Modeling and Analysis of the Car-truck Heterogeneous Traffic Flow Based on Intelligent Driver Model

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ABSTRACT

The traffic flow heterogeneity caused by the different car-following dynamics among the different types of vehicles has drawn increasing attention recently. This paper explores the characteristics of the four types of car-truck car-following combinations, car-following-car (CC), car-following-truck (CT), truck-following-car (TC) and truck-following-truck (TT), and their impacts on traffic flow stability. A heterogeneous traffic flow model based on Intelligent Driver Model (IDM) is proposed and calibrated using the Next Generation Simulation (NGSIM) vehicle trajectory data. Based on the calibrated model, the characteristics of the car-truck heterogeneous traffic flow are evaluated using the linear stability analysis, fundamental diagrams, and shock wave characteristics. The linear stability analysis identifies two critical factors that can influence the stability of the car-truck heterogeneous traffic flow: the stability functions and the proportions of the four types of car-truck combination. Cars and trucks can both stabilize and destabilize the traffic flow depending on the combination type and the equilibrium velocity. Fundamental diagrams of car-truck heterogeneous flow are found to be determined by the distance headways and proportions of the four types of combination. Moreover, the fundamental diagrams of different car-truck combinations converge to several clusters with the same proportion difference between the CC and TT combinations. The slowing-down effect of trucks on shock wave speed in the car-truck heterogeneous traffic flow is also observed in the simulation.

KEYWORDS: CAR AND TRUCK, HETEROGENEOUS TRAFFIC FLOW, INTELLIGENT DRIVER MODEL, CAR-FOLLOWING COMBINATIONS

INTRODUCTION

Heterogeneity is a key characteristic of real-world traffic flow. Although traffic flow theories and models are usually developed first for the homogeneous traffic flow, most of them can be easily converted into their heterogeneous forms. However, the difficulty is the lack of sufficient data to calibrate and explore the heterogeneous models. With the development of the data collection technology in recent years, such as the video-based method and the GPS-based method, traffic information, especially individual vehicle dynamics, can be obtained in greater detail. Many researchers have been using these new data sources to investigate the characteristics of car-truck heterogeneous traffic flow in the past few years. Early researchers focused on investigating the differences between the car and truck driving behavior. Huddart and Lafont (*1*), McDonald et al. (*2*) and Sayer et al. (*3*) compared the headway differences between these two cases: car-following-car and car-following-truck, but their studies did not reach conclusive results regarding with which case had the larger headway. Peeta et al. (*4, 5*) analyzed interactions of cars and trucks in multiple lanes. Highway Capacity Manual presented that trucks occupied more space, had poorer operating capabilities, and created lager gaps than cars in most of situations (*6*). However, these studies (*1-5*) did not recognize that the car-following behavior also depends on the following vehicle type. Ye's study (*7*) first explores the impact of the following vehicle type (car or truck) on traffic flow. He concluded that the four types of car-truck car-following combination should be taken into account in the study of the car-truck heterogeneous traffic flow: the car-following-car (CC), the car-following-truck (CT), the truck-following-car (TC), and the truck-following-truck (TT). Sarvi (*8*) also studied the driving behavior of these three car-following combinations, car-following-car, truck-following-car and car-following-truck. Aghabayk et al. (*9*) further studied the distance headway, time headway, reaction time and car-following threshold variations among the four types of combination. One major limitation of those early studies on car-truck traffic flow characteristics is the lack of modeling of the dynamic traffic flow characteristics. Mason and Woods (*10*) developed the homogeneous Optimal Velocity (OV) car-following model into a heterogeneous traffic flow model to describe the interaction between cars and trucks. The derived OV heterogeneous car-following model is as follows,

$$\frac{d^2 x_n(t)}{dt^2} = \lambda_n \left(U_n \left(x_{n-1} - x_n \right) - v_n \right) \tag{1}$$

where $x_n(t)$ and $v_n(t)$ respectively denote the location and velocity of the vehicle n at time t, $x_{n-1}(t)$ denotes the location of the vehicle n-1 (the preceding vehicle of vehicle n) at time t, λ_n denotes the sensitivity parameter of the vehicle n, $U_n\left(x_{n+1} - x_n\right)$ denotes the optimal velocity function the vehicle n wishes to take, and it is the function of the headway of vehicle n.

The heterogeneous Optimal Velocity model is used to conduct the comparison of the car-following dynamics between homogeneous and heterogeneous vehicles and it is found that trucks may

dampen congestion waves. Because of the complication in their multi-species formulations, it is difficult to analyze individually how different car-truck following combinations can affect traffic flow. In this study, based on Intelligent Driver Model (IDM), we developed a heterogeneous model with the different sub-models for each car-truck following combination. To calibrate the new model, the model parameters for each car-truck car-following combination are calibrated separately using the car-following data extracted from the I-80 NGSIM (Next Generation Simulation) data for each combination. Based on the calibrated model, we study the three characteristics of the car-truck heterogeneous traffic flow, the linear stability, fundamental diagrams, and shock wave.

METHODOLOGY

An IDM-based Car-following Model for the Car-truck Heterogeneous Traffic Flow

Treiber et al. proposed Intelligent Driver Model in 2000 (*11*) for the homogeneous traffic flow. This model is a widely explored car-following model *(12-14)*, and its formulation is as follows,

$$\begin{cases} \dfrac{d^2x_n(t)}{dt^2} = a\left[1-\left(\dfrac{v_n(t)}{V}\right)^{\delta}-\left(\dfrac{S(v_n(t),\Delta v_n(t))}{\Delta x_n(t)-l}\right)^2\right] \\ S(v_n(t),\Delta v_n(t)) = s^0 + s^1\sqrt{\dfrac{v_n(t)}{V}} + \tau v_n(t) - \dfrac{v_n(t)\cdot\Delta v_n(t)}{2\sqrt{ab}} \end{cases} \tag{2}$$

where a is the maximum acceleration, V is the desired velocity, δ is the acceleration exponent, $S(\cdot)$ is the desired minimum gap, s^0 and s^1 are the jam distances, τ is the safe time headway, b is the desired deceleration, l is the leading vehicle length, and $\Delta v_n(t) = v_{n-1}(t) - v_n(t)$ is the removal rate of the vehicle n to its preceding vehicle n-1.

We develop the homogeneous IDM to its heterogeneous form by giving subscripts to the model parameters. The proposed heterogeneous IDM formulates the four different car-truck car-following combinations as follows,

$$\begin{cases} \dfrac{d^2x_n(t)}{dt^2} = a_n\left[1-\left(\dfrac{v_n(t)}{V_n}\right)^{\delta_n}-\left(\dfrac{S_n(v_n(t),\Delta v_n(t))}{\Delta x_n(t)-l_n}\right)^2\right] \\ S_n(v_n(t),\Delta v_n(t)) = s_n^0 + s_n^1\sqrt{\dfrac{v_n(t)}{V_n}} + \tau_n v_n(t) - \dfrac{v_n(t)\cdot\Delta v_n(t)}{2\sqrt{a_n b_n}} \end{cases} \tag{3}$$

where all the parameters a_n, δ_n, V_n, s_n^0, s_n^1, τ_n and b_n have four alternatives. Taking a_n as an example, a_n can be a_{cc}, a_{ct}, a_{tc} and a_{tt}. The leading vehicle length l has two alternatives, l_c and l_t.

In the homogeneous traffic flow, all vehicles at the equilibrium state have zero acceleration, the same distance headway and the same velocity; while at the equilibrium state, all vehicles in the

heterogeneous traffic flow still have zero acceleration and the same velocity, but their distance headways vary for different vehicles which can be described using the following equations:

$$v_n = v^* ,\ \dot{v}_n = 0 \text{, and } h_n = h_n^* \tag{4}$$

where v^* is the equilibrium velocity which is the same for all vehicles, h_n^* is the corresponding equilibrium headway of the vehicle n, and h_n^* varies among the vehicles. Substituting (4) into the heterogeneous IDM (3) yields,

$$\begin{cases} 1-\left(\dfrac{v^*}{v_0}\right)^{\delta} - \left(\dfrac{\hat{s}(v^*,0)}{h_{IDM}^* - l}\right)^2 = 0 \\ \hat{s}(v^*,0) = s^0 + s^1\sqrt{v^*/v_0} + \tau v^* \end{cases} \tag{5}$$

Rewriting the above equation yields,

$$h^* = \frac{s_0 + s_1\sqrt{v^*/v_0} + \tau v^*}{\sqrt{1-\left(v^*/v_0\right)^{\delta}}} + l \tag{6}$$

In the car-truck heterogeneous flow, h^* has the four alternatives, h_{cc}^*, h_{ct}^*, h_{tc}^* and h_{tt}^* which correspond to the four types of combinations. It should be noticed that the vehicle length l in h^* only depends on the leading vehicle type, namely, $l = l_c$ in h_{cc}^* and h_{tc}^*, and $l = l_t$ in h_{ct}^* and h_{tt}^*.

Then, the fundamental diagram of the heterogeneous IDM for the equilibrium state can be derived based on Equation (6). Assume a given heterogeneous traffic flow on a single lane contains N vehicles. The length of the entire traffic flow in the equilibrium state is:

$$L_{total} = \sum_{n=1}^{N} h_n^* \tag{7}$$

In the car-truck heterogeneous traffic flow, L_{total} has the following formula:

$$L_{total} = N_{cc}h_{cc}^* + N_{ct}h_{ct}^* + N_{tc}h_{tc}^* + N_{tt}h_{tt}^* \tag{8}$$

where N_{cc}, N_{ct}, N_{tc} and N_{tt} are the numbers of the CC, CT, TC and TT combinations in the car-truck heterogeneous traffic flow. Hence, the density of traffic flow can be calculated as:

$$k = \frac{N}{N_{cc}h_{cc}^* + N_{ct}h_{ct}^* + N_{tc}h_{tc}^* + N_{tt}h_{tt}^*} \tag{9}$$

Rewriting equation (9) in the following form:

$$k = \frac{1}{P_{cc}h_{cc}^* + P_{ct}h_{ct}^* + P_{tc}h_{tc}^* + P_{tt}h_{tt}^*} \tag{10}$$

where P_{cc}, P_{ct}, P_{tc} and P_{tt} are the proportions of the CC, CT, TC and TT combinations. Thus, the fundamental diagram of the car-truck heterogeneous traffic flow has the following relationship:

$$q = \frac{v^*}{P_{cc}h_{cc}^* + P_{ct}h_{ct}^* + P_{tc}h_{tc}^* + P_{tt}h_{tt}^*} \tag{11}$$

where q is the flow rate of the car-truck heterogeneous traffic flow in space.

Equation (11) indicates that the distance headways and proportions of the four types of combination determine the fundamental diagram of the car-truck heterogeneous traffic flow.

Linear Stability Criterion of the Car-truck Heterogeneous Traffic Flow

The linear stability analysis investigates the perturbation propagation characteristic of a vehicle platoon by adding a small perturbation on the first vehicle of the platoon (*13, 15*). The stability criterion of IDM can be derived by following the general stability criterion summarized by Wilson and Ward (*13*). The derivation of the detailed stability criteria of IDM requires the partial differentials of the velocity, headway and velocity difference between the leading and following vehicles with respect to the equilibrium state of IDM.

$$f_v = -a\delta\left(\frac{v^*}{V}\right)^{\delta-1} - \frac{2aS^*}{\left(h^*-l\right)^2}S_v,\ f_{\Delta v} = -\frac{2aS^*}{\left(h^*-l\right)^2}S_{\Delta v},\ f_h = 2a\left(S^*\right)^2\left(h^*-l\right)^{-3} \tag{12}$$

where $S^* = S(v^*,0,h^*)$. S_v, $S_{\Delta v}$ and S_h are respectively the partial differences of the desired minimum gap function with respect to the v_n, Δv_n and Δx_n at the equilibrium state $(v^*,0,h^*)$. Hence, according to the general stability criterion introduced by Wilson and Ward (*13*), the stability criterion of IDM is as follows,

$$f_{\Delta v}f_v + f_h - \frac{f_v^2}{2} < 0 \tag{13}$$

Substituting equation (12) into (13) yields the stability criterion of IDM,

$$\frac{2\left(S^*\right)^2}{\left(h^*-l\right)^3} + \frac{2aS^*S_{\Delta v}}{\left(h^*-l\right)^2}\left(\delta\left(\frac{v^*}{v_0}\right)^{\delta-1} + \frac{2S_vS^*}{\left(h^*-l\right)^2}\right) - \frac{a}{2}\left(\delta\left(\frac{v^*}{v_0}\right)^{\delta-1} + \frac{2S^*}{\left(h^*-l\right)^2}\right)^2 < 0 \tag{14}$$

The derivation of the linear stability criterion of the car-truck heterogeneous traffic is more complicated. Ward (*16*) presented the general formula of the linear stability criterion of the heterogeneous traffic flow as follows,

$$\sum_n\left[\left(f_{n,\Delta v}\cdot f_{n,v} + f_{n,h} - f_{n,v}^2/2\right)\cdot\left(\prod_{j\neq n} f_{j,h}\right)^2\right] < 0 \tag{15}$$

where $f_{n,v}$, $f_{n,\Delta v}$, $f_{n,h}$ are the partial differences of the car-following model adopted by the vehicle n with respect to v, Δv and Δx at the equilibrium state where $v_n = v^*$, $\Delta v_n = 0$ and $\Delta x_n = h_n^*$.

However, Equation (15) cannot directly reflect the proportion information of the different combinations, so we rewrite it as the following form using the induction method:

$$\sum_{i=1}^{Q}\left[NP_i\left(f_{i,\Delta v}\cdot f_{i,v} + f_{i,h} - f_{i,v}^2/2\right)\cdot\left(f_{i,h}\right)^{2NP_i-2}\prod_{\substack{j\neq i\\ j=1}}^{Q}\left(f_{j,h}\right)^{2NP_i}\right] < 0, \tag{16}$$

where Q is the number of the combinations in a given car-truck heterogeneous traffic flow, N is the total number of vehicles in the traffic flow, N_i is the total number of the combination type i in the platoon, $P_i = \frac{N_i}{N}$ is the proportion of the combination type i in total, and $\sum_{i=1}^{M} P_i = 1$. In the car-truck heterogeneous traffic flow, $Q = 4$ and i has four alternatives: CC, CT, TC and TT.

Equation (16) can be further simplified as follows,

$$\sum_{i=1}^{Q}\left[P_i\frac{f_{i,\Delta v}\cdot f_{i,v} + f_{i,h} - f_{i,v}^2/2}{f_{i,h}^2}\right] < 0 \tag{17}$$

In Equation (17), two parts determine the stability criterion of the car-truck heterogeneous traffic flow: the fraction part $\frac{f_{i,\Delta v}\cdot f_{i,v} + f_{i,h} - f_{i,v}^2/2}{f_{i,h}^2}$ providing the stability characteristic of the combination i and the proportion part P_i of the combination i. We define the fraction part as the Stability Function (*SF*) of the combination,

$$SF_i(f_{i,v}, f_{i,\Delta v}, f_{i,h}) = \frac{f_{i,\Delta v}\cdot f_{i,v} + f_{i,h} - f_{i,v}^2/2}{f_{i,h}^2} \tag{18}$$

where SF_i is the stability function of the combination type i. In addition, comparing the stability criterion of IDM (14) with equation (18), it can be found that *SF* has the same sign as (14), so *SF* also can be used to judge the stability of the car-following combination i. The stability effects of combination type i denoted as SF_i in the car-truck heterogeneous traffic flow can be written as follows,

$$SF_i = \frac{\left(h_i^* - l_i\right)^6}{4a_i^2\left(S_i^*\right)^4}\cdot\left[\frac{2\left(S_i^*\right)^2}{\left(h_i^* - l_i\right)^3} + \frac{2a_iS_i^*S_{i,\Delta v}}{\left(h_i^* - l_i\right)^2}\left(\delta_i\left(\frac{v^*}{V_i}\right)^{\delta-1} + \frac{2S_{i,v}S_i^*}{\left(h_i^* - l_i\right)^2}\right) - \frac{a_i}{2}\left(\delta_i\left(\frac{v^*}{V_i}\right)^{\delta_i-1} + \frac{2S_i^*}{\left(h_i^* - l_i\right)^2}\right)^2\right] \tag{19}$$

Moreover, we define the left part of equation (17) as the stability function F of the car-truck heterogeneous traffic flow. Thus, in the car-truck heterogeneous traffic flow, there is,

$$F = P_{cc}\cdot SF_{cc} + P_{ct}\cdot SF_{ct} + P_{tc}\cdot SF_{tc} + P_{tt}\cdot SF_{tt} \tag{20}$$

When $F < 0$, the car-truck heterogeneous traffic flow is stable. Equation (20) indicates that the proportion of the car and the truck is not the only deciding factor of the stability of heterogeneous traffic flow, and the stability function of each car-truck following combination is also significant. Furthermore, on a ring road, the stability function of the car-truck heterogeneous traffic flow and the proportions of four types of combination have the following relationship:

$$\begin{cases} F_l = P_{cc} \cdot SC_{cc} + P_{ct} \cdot SC_{ct} + P_{tc} \cdot SC_{tc} + P_{tt} \cdot SC_{tt} \\ P_{cc} + P_{ct} + P_{tc} + P_{tt} = 1 \\ P_{ct} = P_{tc} \end{cases} \tag{21}$$

Model Calibration and Experimental Design

The car-following model calibration is a nonlinear optimization problem. In this study, we use Genetic Algorithm (GA) to solve this nonlinear optimization problem to obtain the optimal parameters. We also adopt the Theil's *U* function as the objective function as suggested in several existing studies (*17, 18*).

$$F_{objective} = \frac{\sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(y_m^{real} - y_m^{sim}\right)^2}}{\sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(y_m^{real}\right)^2} + \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(y_m^{sim}\right)^2}} \tag{22}$$

where y_m^{real} is the real data, y_m^{sim} is the simulation result from the model, and m=1, …, M is the number of the data sample.

To calibrate car-following models, we use the NGSIM vehicle trajectory data collected on I-80 San Francisco, California on April 13, 2005 (*19*). Since the data includes lane change and large vehicle gaps, we use the following criteria to extract the car-following datasets of the four combinations separately from NGSIM data.

a. Each car-following group contains two vehicles (car-following pair). Car-following group with more than three vehicles will be divided into multiple two-vehicle groups.
b. Each pair of car-following vehicles is formed and decomposed based on two spacing thresholds, the engaging threshold DE = 130 ft (39.62 m) and the disengaging threshold DD = 150 ft (45.72 m). These values are determined based on critical density at the experimental site. Using two thresholds instead of one threshold can avoid unnecessary frequent grouping and ungroup of car-following vehicle pairs caused by small fluctuation of spacing value around a single threshold.
c. If a vehicle group decomposes due to spacing increase, lane changing or reaching the end of the segment, it will not be exported as a valid sample.

The final datasets include the 477712, 25844, 16471, and 10105 trajectory points for the CC, CT, TC, and TT combinations respectively. The calibrated car-following models are also inspected using several error indexes including Mean Error (ME), Mean Absolute Error (MAE) and Mean Absolute Relative Error (MARE) as follows,

$$ME = \frac{\sum_{m=1}^{M}(y_m^{real} - y_m^{sim})}{M} \quad (23)$$

$$MAE = \frac{\sum_{m=1}^{M}\left|y_m^{real} - y_m^{sim}\right|}{M} \quad (24)$$

$$MARE = \frac{\sum_{m=1}^{M}(\left|y_m^{real} - y_m^{sim}\right| / y_m^{real})}{M} \quad (25)$$

where m indicates the m^{th} sample, M is the total number of samples, y_m^{real} is the m^{th} real data sample (y can be the acceleration, velocity or position), and y_m^{sim} is the m^{th} simulation data sample.

To investigate the stability and characteristics of the car-truck heterogeneous traffic flow, a ring-road numerical simulation is conducted by placing 100 vehicles on a single-lane, flat, and ring road, with the first vehicle following the end vehicle. In the simulations, the initial state of the traffic flow is equilibrium state, add a small perturbation on the first vehicle, and then observe the perturbation development in the platoon. Many studies adopted this simulation method to explore the fundamental diagram (*20, 21*), shock wave (*21, 22*), linear stability (*13, 23*), and other properties (*24*) of traffic flow.

RESULTS ANALYSIS

Preliminary Data Analysis

Some preliminary analysis is conducted to obtain the general characteristics of the different car-truck car-following combination types. First, the gap versus velocity relationship for each combination type is explored. The CC combination has the smallest average gap, and the TT combination has the largest average gap for the same velocity. When v = 17 m/s, the CT combination has the same gap with the TC combination. The CT combination has a larger gap than the TC combination before v = 17 m/s, while the TC combination has a larger gap after v = 17 m/s. Second, the desired velocities of cars and trucks are also studied. The statistical results show that the truck has the maximum velocity 18 m/s, and the car has the maximum velocity 30 m/s. The maximum velocity of the CT combination is around 18m/s which is restricted by the maximum velocity of the leading truck. The maximum velocity found for the CT combination is around 23 m/s with the combined impacts of the slow-moving follower truck and fast-

moving leader car. The CC and TT combinations respectively had the maximum velocities of 30 m/s and 16 m/s. Third, the significant difference in response time for the different vehicle types suggested by (*25*) can be measured using the method introduced by Aghabayk et al. (*9*). The average response time of the truck drivers is found to be 2.1 s, and the average response time of the car drivers is around 1.3 s.

Calibration and Evaluation Results

Table 1 lists the calibration results of the heterogeneous IDM. The calibrated desired velocities of the four combinations are similar to the preliminary statistical results. The TT combination has the largest jam spaces (s_n^0 and s_n^1); while the CC combination has the smallest jam spaces. The order of the safe time headway τ_n is CC < CT < TC < TT. Cars have larger deceleration b_n than trucks. The acceleration exponents are same for all four types of combination. The values of the error indexes of acceleration, velocity, and location for each combination type can also be found in Table 1. The acceleration simulation errors (MARE) for the four combinations are all less than 15%, and the velocity and location simulation errors (MARE) are all less than 10%. Therefore, the calibration results can reflect the car-following behavior characteristics of the four combinations in the I-80 NGSIM vehicle trajectory data.

TABLE 1 Calibration and evaluation results of the four combinations

	Variables	CC	CT	TC	TT
Calibration Results	a_n (m/s^2)	1.01	1.03	0.78	0.74
	V_n (m/s)	27	19.3	20.6	17.7
	s_n^0 (m)	0.85	1.35	1.11	1.53
	s_n^1 (m)	0.19	0.27	0.12	0.36
	τ_n (s)	1.2	1.4	1.8	2.0
	b_n (m/s^2)	2.26	2.12	1.70	1.61
	δ_n	4	4	4	4
Acceleration Evaluation	ME	0.84	0.28	-0.03	0.13
	MAE	1.17	1.67	1.24	1.16
	MARE	0.11	0.13	0.12	0.1
Velocity Evaluation	ME	-0.25	0.15	-0.23	-0.04
	MAE	1.31	1.62	1.2	0.75
	MARE	0.08	0.09	0.07	0.05
Location Evaluation	ME	2.44	3.00	-1.93	1.23
	MAE	9.71	9.6	5.75	5.61
	MARE	0.05	0.06	0.03	0.04

Linear Stability Analysis

The key focus of the linear stability analysis is to investigate the impacts of the two critical factors on the stability of the car-truck heterogeneous traffic flow: the individual stability and the proportions of the four types of combination.

Individual Stability of the four combinations

The stability function SF_i for each combination type i is used to evaluate its individual stability effect on the car-truck heterogeneous traffic flow. FIGURE 1 displays the relationship between SF versus the equilibrium velocity for each combination type. The relative order of the SF for each combination type has the following three cases.

- CT≈CC>TC>TT. This case happens before v^* = 9 m/s. In this case, the CT combination is the most unstable combination and the TT combination is the most stable combination. The CC combination effect is close to the CT combination. The truck as the following vehicle is more stable than the car as the following vehicle.
- CC>CT>TC>TT. This case occurs in the middle range of velocity from 9 m/s to 11 m/s, and the CC combination becomes the most unstable case. The truck as the leading vehicle in the combination is more stable than the car as the leading vehicle, and the truck as the following vehicle in the combination is more stable than the car as the following vehicle.
- CC>TC>CT>TT. This case occurs when the velocity is more than 11 m/s. In this case, the CT combination is more stable than the TC combination. The truck as the leading vehicle is more stable than the car as the leading vehicle.

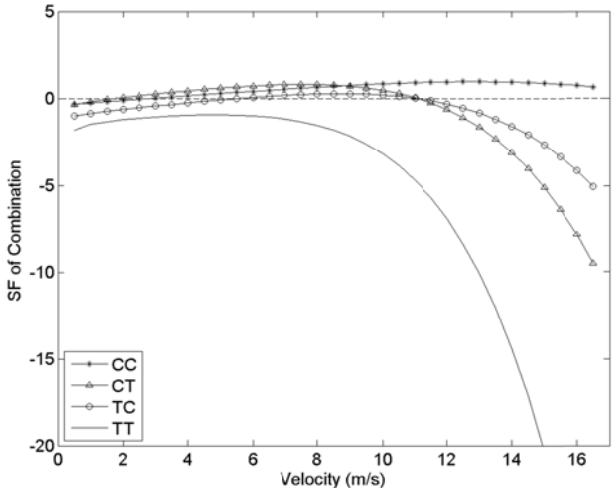


FIGURE 1 Stability functions of the four combinations with respect to the uniform velocity.

From the above analysis, the following conclusion can be drawn: the stability effect of cars and trucks depends on their leading or following status in car-following and the equilibrium velocity of the car-truck heterogeneous traffic flow. This conclusion is more sophisticated than the previous studies (*10, 26*) in which the truck or car only has one single effect (stabilize or destabilize) on the traffic flow.

In the following ring-road simulation, to demonstrate the different effects of the four combinations on traffic flow, we fill the ring road with 30 vehicles alternating between cars and trucks followed by 10 cars, 30 trucks and 30 cars. Therefore, the proportions of the CC, CT, TC and TT combinations are 39%, 16%, 16% and 29%. Set the equilibrium velocity as 4 m/s. In this case, the stability function of the car-truck heterogeneous traffic flow F = -0.410 indicating that the traffic flow is stable. The stability functions of the four combinations are SF_{cc} = 0.28, SF_{ct} = 0.43, SF_{tc} = -0.83, and SF_{tt} = -1.57, which means that the CC and CT combinations can amplify the perturbation, the TC and TT combinations will suppress the perturbation, and the car-truck alternating can decrease the perturbation ($SF_{ct} + SF_{tc} < 0$). The shock wave peak (See Figure 2) fluctuates and descends at first (the car-truck alternating part), ascends after the Vehicle No.31 (the CC combination part), and descends after the Vehicle No.41 (the TT combination part), and ascends slightly again after the Vehicle No.71 (the CC combination part). Thus, the simulation results are consistent with the analytical analysis.

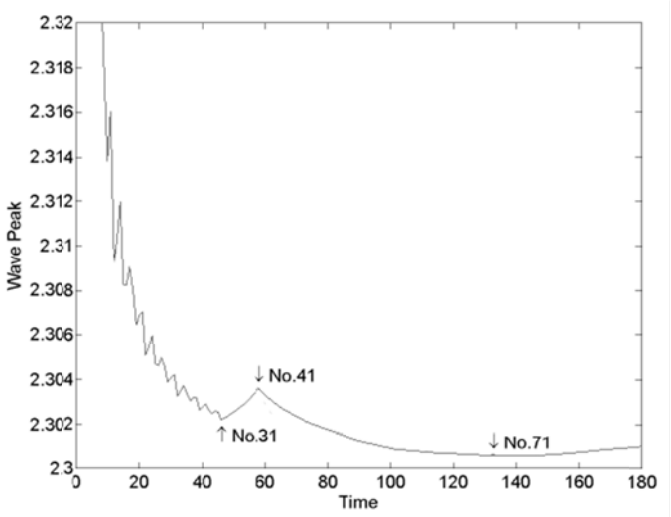
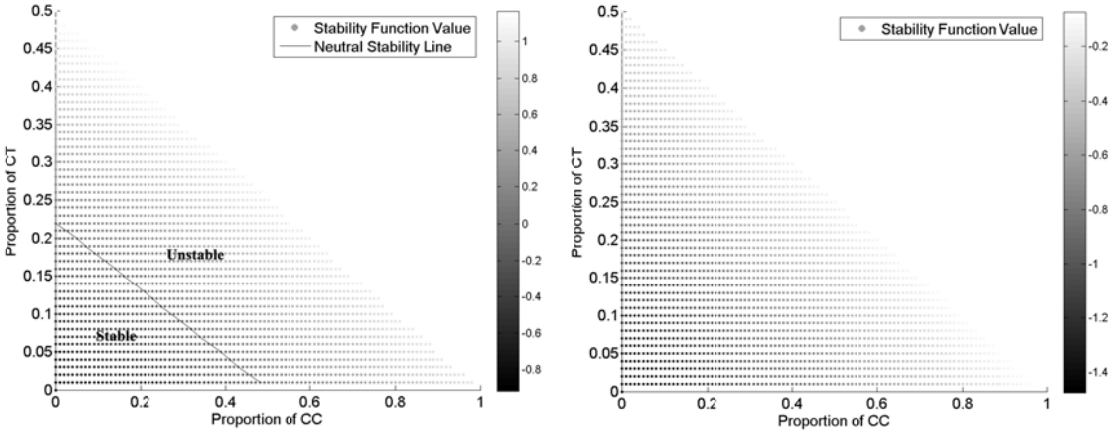


FIGURE 2 The profile of the shock wave peak.

Impact of the proportions of the four combinations on stability

As shown in Equation (21), the stability function of the car-truck heterogeneous traffic flow is a weighted average of the individual values SF_{cc}, SF_{ct}, SF_{tc}, and SF_{tt} according to the proportions of the car-truck combination types. Take the case v^* = 6 m/s as an example and find the corresponding SF values of the four combinations from FIGURE 1. Figure 3 (a) is the scatter plot of the stability function

values of the car-truck heterogeneous traffic in the space of the CC and CT proportions (According to Equation (21), the proportions of CC and TT can determine the proportions of CT and TC). The dots in FIGURE 3 (a) are the stability function values, and the line is the neutral stability line connecting all the zero stability function values. The neutral stability line splits the quadrant into two areas, the unstable area above the line and the stable area under the line. (0, 0.22) and (0.5, 0) are the two critical points. When the proportion of the CC combination is more than 0.5 or the CT combination is more than 0.22, the traffic flow is always unstable regardless of the proportions of the other three combinations. However, when v^* = 1 m/s as shown in FIGURE 3 (b), the four *SF* are all negative, which means all four combinations are stable. Thus, no neutral stability line exists in this case since the overall stability function values are always less than zero.

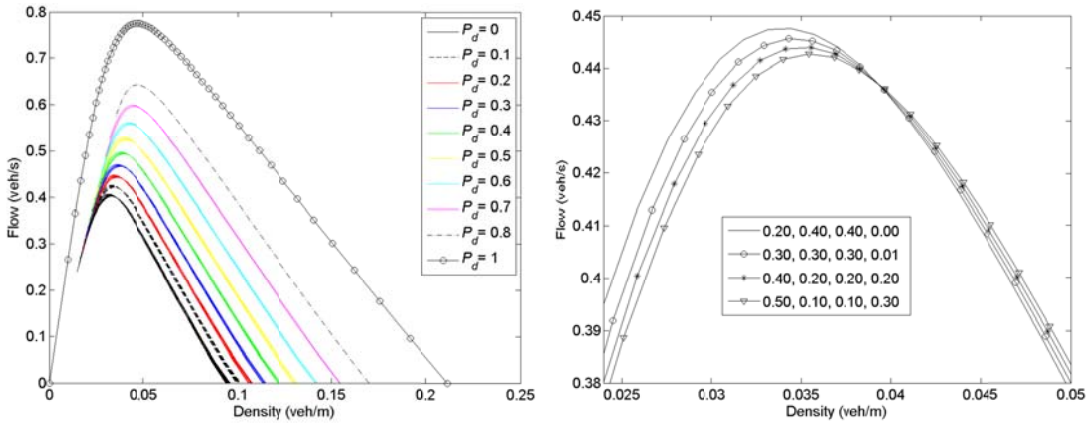


(a) (b)

FIGURE 3 Scatter plots of the stability function of the car-truck heterogeneous traffic flow. (a) The case v^* = 1 m/s. (b) The case v^* = 6 m/s.

Fundamental Diagram Analysis

We explore the fundamental diagram characteristic of the car-truck heterogeneous traffic flow based on equation (11). When examining the fundamental diagram curves for all proportion combinations of the CC, CT, TC and TT, we find that the difference between the CC and TT combinations $P_d = P_{cc} - P_{tt}$ is a critical factor to determine the shape and location of the overall fundamental diagram. It is found that fundamental diagrams with similar P_d values locate close to each other on the flow-density plot as shown in FIGURE 4 (a). FIGURE 4 (b) illustrates the cluster for the case $P_d = 0.2$ and the numbers beside the legends are the proportions of the four combinations CC, CT, TC and TT respectively. In FIGURE 4 (b) the curves have an intersection, and the flow decreases with the increase of P_{cc} before the intersection and increases after the intersection. Furthermore, the flow rate and critical density both increase with the increasing of P_d, which reveals that the most unstable point of the car-truck heterogeneous traffic flow moves towards the high flow and high density area with the increasing of P_d.

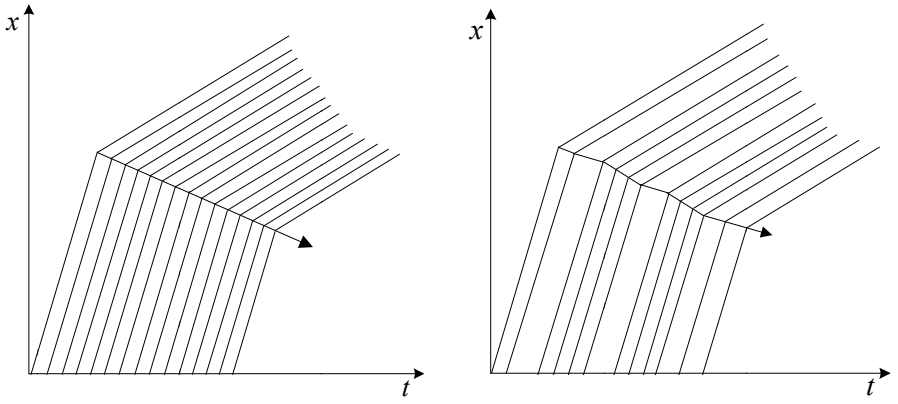


(a) (b)

FIGURE 4 Fundamental diagrams of the car-truck heterogeneous traffic flow. (a) Fundamental diagrams. (b) Fundamental diagram cluster at P_d =0.2.

Shock Waves Analysis

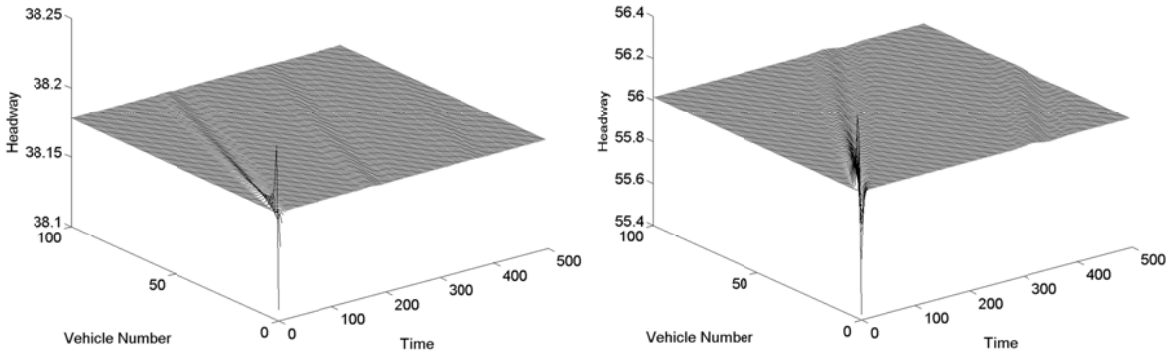
The propagation speed of the shock wave is not uniform through the platoon due to the response time differences between cars and trucks. FIGURE 5 (a) and (b) respectively demonstrate the shock wave propagations in the pure car traffic flow and the car-truck heterogeneous traffic flow from the trajectory viewpoint. The arrow indicates the propagation trajectory of a perturbation which gives the shock wave boundaries between two different states of traffic flow. The slope of the shock wave in the pure car traffic flow is the same during propagation. However, the slope changes when going through the car-truck heterogeneous traffic flow and the average slope decreases in the car-truck heterogeneous traffic flow because the truck needs more time to react to the speed change of its preceding vehicle. Thus, the shock wave travels faster in the pure car traffic flow than in the car-truck heterogeneous traffic flow.



(a) (b)

FIGURE 5 Propagation of the shock wave from the trajectory viewpoint. (a) The pure car traffic flow. (b) The car-truck heterogeneous traffic flow.

FIGURE 6 illustrates the shock wave propagation observed in a simulation of perturbation propagation in both pure car and mixed car-truck flow. The traffic flow in FIGURE 6 (a) is the pure car traffic flow, and the traffic flow in FIGURE 6 (b) is the car-truck heterogeneous traffic mixed by 70 cars and 30 trucks. Giving the same perturbation to the two types of traffic flow at the same place, the shock wave takes around 250 s to propagate to the last vehicle in the pure car traffic flow; while in car-truck heterogeneous flow, it takes around 400 s. The simulations illustrate that the car-truck heterogeneous traffic flow can slow down the shock wave.



(a) (b)

FIGURE 6 Simulation of the propagation speed of the shock wave. (a) The pure car traffic flow. (b) The car-truck heterogeneous traffic flow.

CONCLUSION

The traffic flow mixed by cars and trucks is the typical heterogeneous traffic flow in real traffic. In this paper, the focus is the analysis of the impacts of the four types of combination, car-following-car (CC), car-following-truck (CT), truck-following-car (TC) and truck-following-truck (TT), using heterogeneous car-following models. We extend the original Intelligent Driver Model (IDM) to a heterogeneous form and use the NGSIM data to calibrate model parameters. The evaluation results show that the calibrated car-following models can reflect the car-following dynamics of the four combinations in the car-truck heterogeneous traffic flow. Based on the calibrated car-following model, we explore the three characteristics of the car-truck heterogeneous traffic flow: the linear stability, fundamental diagrams and shock wave.

We derive the linear stability criterion of the car-truck heterogeneous traffic flow and validate it using the ring-road simulations. Two critical factors, the individual stability function and the proportions of the four combinations, are found to determine the stability of the car-truck heterogeneous traffic flow. Furthermore, the individual stability function varies with the increasing of equilibrium velocity; especially with the increasing of equilibrium velocity, TC becomes less stable than CT. In addition, cars

and trucks can both stabilize and destabilize the car-truck heterogeneous traffic flow and their effects depend on their roles (leading vehicle or following vehicle) in car-following and the equilibrium velocity of the car-truck heterogeneous traffic flow. This conclusion is more sophisticated than the previous studies (*10, 26*) in which the truck or car only has one single effect (stabilize or destabilize) on the traffic flow. We also investigate the impact of the proportions of the four combination types by simulation. The results show that the proportions of the four combinations may not have significant influence on the stability of traffic flow with low equilibrium velocity (e.g. on an extremely congested road); however, in most cases, the neutral stability lines can be found for the car-truck heterogeneous traffic flow.

Fundamental diagrams of the car-truck heterogeneous traffic flow are determined by the distance headways and the proportions of the four types of combination. The simulations further reveal that the fundamental diagrams with the same proportion difference between the CC and TT combinations cluster together. The flow rate and the critical density increase with the increasing of the proportion difference between the CC and TT. Finally, the shock wave analysis indicates that trucks can slow down the propagation speed of the shock wave due to their longer response times.

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